Notes on *al-Bīrūnī on Transits*

G. J. Toomer - Oxford

One of the most important contributions in recent years to our knowledge of the transmission of Indian astronomy to the Islamic world and to early Islamic astronomy is the book *al-Bīrūnī on Transits*, translated by Mohammad Saffouri and Adnan Ifram, with a commentary by E. S. Kennedy, American University of Beirut, 1959. In the course of reviewing this book I made a number of additions and corrections, which I now offer to future readers of it in the hope that their understanding of the very difficult text may be helped somewhat. I cannot claim to have removed all of the many difficulties which remain after Kennedy's heroic ground-clearing operation: I have confined my notes to those places where I could correct or materially supplement the translation and commentary with certainty or at least some plausibility. Like the translators, I have not had access to the manuscript, but have used the Arabic text printed in *Rasā'ilul-Bīrūnī*, Osmania Oriental Publications Bureau, Hyderabad-Dn, 1948. This is a corrupt one, and I have freely suggested emendations. I have ordered my notes according to the page and line numbering of that text, exactly as the translators did. I refer to Kennedy's commentary by the page number of the translation. All references are to the latter work where there is no other indication. ' B. ' stands for al-Bīrūnī throughout. For the elucidation of other references see the bibliography at the end of this article.

I am very grateful to Dr. S. M. Stern and Dr. David Pingree for their help. Some of Dr. Pingree's notes have been incorporated whole, and are marked as his.

2:4 *Instead*: Translate: ' Except that '. In this section B. is dealing with those configurations in which the western motion *does* have a part, i.e. anything to do with the observer's horizon. — **2:17-18** ' in latitude ' means ' when it has a latitude ', i.e. when its latitude is not zero. — **3:18-4:1** Signs equidistant from a solstice were called, among other things, *antiscia* in Greco-Roman astrology. The locus classicus on this is Firmicus II 29. (Pingree) — **4:3** *doubles*: Read *mailāhumā* for *mṭl'hm'* and translate ' declinations ' (dual) cf. 5:4. — **4:12** The note on p. 125 is misplaced, and should come at 4:17. In 4:12-16 B. is still dealing with pairs of signs equidistant from a solstitial point. Cf. 4:5 and 4:16. — **4:12** There are several references in Vettius Valens to ἰσοδυναμοῦντα ζῴδια, but they are nowhere defined explicitly. However in III 7 (Kroll p. 142,27,

cf. p. 257,35) it seems probable that they are equated with the ἰσανάφορα (signs of equal rising times), which would agree with what is said here. But it is known that the text as printed by Kroll is not the complete Valens, so B. may be reproducing here — at third hand — a missing portion of the text. The same may be said of 4:13-15 and 5:7-9. — **4:16** *aspect*: Though this section remains obscure, it is clearly dealing with βλέποντα ζῴδια, which here, as in Ptolemy, seem to be equated with the
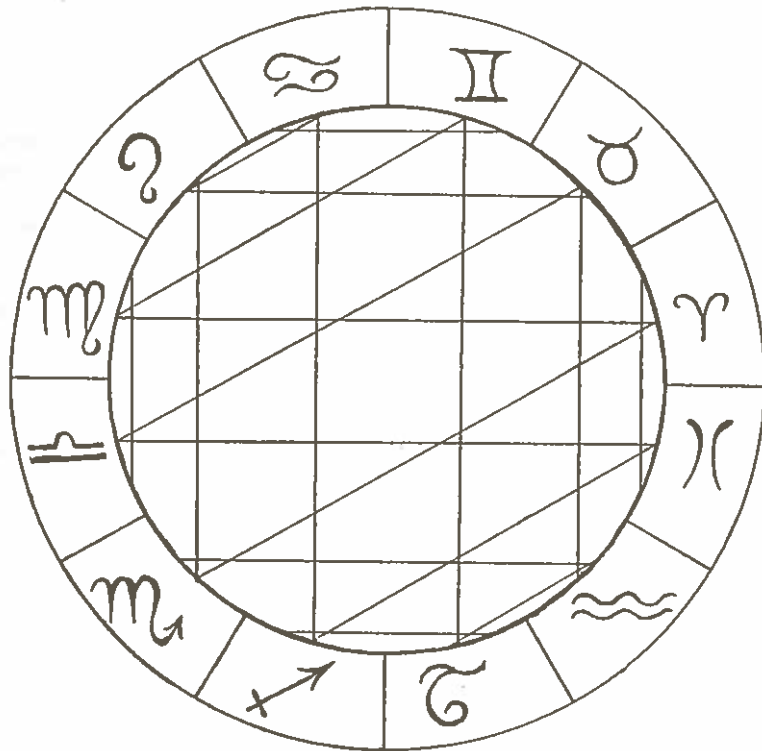
Fig. I

ἰσοδυναμοῦντα. See Bouché-Leclercq pp. 159-161. — **5:3** *some of them call the elevated one commandant* etc. Ptolemy is one of these: *Tetrabiblos* I 15 (Boll-Boer p. 37). — **5:5** *their days (are) also equal*: this is patently false, and in fact the Arabic is *mutakāfi'an*, which means 'corresponding', i.e. the day of one is equal to the night of the other. Translate 'corresponding in their days'. — **5:6** This is indeed found in Vettius Valens, e.g. Kroll 238,12 ff. — **5:6-6:5** The translation is totally unintelligible because 'corresponding in course' has been used to translate *both muttafiqa fi ṭ-ṭarīqa and muttafiqa fi l-minṭaqa*, which are quite different concepts for B. Three types of corresponsion are being distinguished: (1) The

signs equidistant from a solstice (connected by horizontal lines in my Fig. I). These have equal hours of daylight. Ptolemy calls them ἰσοδυναμοῦντα. B. calls them *muttafiqa fi ṭ-ṭarīqa* in the Tafhīm (p. 228), and following Wright there I shall translate that 'corresponding in course'. (2) The signs equidistant from an equinox (connected by vertical lines in Fig. I). These have equal ascensions. Abū Ma'šar (5:12-13) called them *muttafiqa fi l-minṭaqa*, which I shall translate 'corresponding in zone'. (3) The signs which are houses of the same planet (connected by sloping lines in Fig. I). In the case of Cancer and Leo, the former is the house of the moon, the latter of the sun. For the doctrine see Bouché-Leclercq p. 182 ff. To make clear who calls what by what name, I set them out in a little table:

	(1)	(2)	(3)
Author of the *Bizīdhaj* (4·12-5:8)	c. in strength	c. in ascensions	c. in course
Abū Ma'šar (5:10-13)	c. in strength *or* potent	c. in zone	c. in course
al-Saifī (5:14-16)	c. in course	*not known but criticised Abū Ma'šar's* 'c. in zone'	*not known*
B.'s recommendations (6:1-5)	c. in course	c. in times *or* potent	c. in strength

The translation should be changed at the following places: **5:12-13** Translate 'corresponding in *zone*, and he left the name of the third type unchanged'. By the latter statement B. means that Abū Ma'šar, like the *Bizīdhaj*, called (3) 'corresponding in course'. This is explicitly stated in *Tafhīm* p. 228. — **5:16** Translate 'corresponding in *zone*, and he ascribed it to ignorance of the *zones*'. The note on 5:16 is misleading. It is true that *minṭaqat* (vocalised thus rather than *manṭaqat*) *al-burūj* is used by B. (e.g. 6:6) interchangeably with *falak al-burūj* for the zodiac. But that does not mean that *minṭaqa* ('belt') is in general a synonym of *falak* ('sphere'), and in this work at least B. uses the word *minṭaqa* (without *al-burūj*) in only one other passage (97:12-98:9, on which see my note), where it is certainly not a synonym of *falak*. In 5:16 the meaning cannot be determined, and it is best to translate by a non-committal word. — **6:3** *and ... one method*: Translate 'we draw them along a single course' (*ṭarīqa*), i.e. we consider that they move along the same apparent path through the heavens. — **6:6** *the distance to*: Read *al-munsāqa* for *'lms'fh* and translate 'which is carried along'. — **6:17** Translate 'that all that

is necessary for the longitudinal type of transit is going in front and falling behind only ', i.e. actual passing of one body by another, whereas in 6:14-16 he has been talking of one body passing the place where another *had previously been*. — **7:12** Translate: ' *the* locality '. — **7:12-13** ' coming forward ' and ' going away ' are translations of the Greek technical astrological terms ἐπαναφορά and ἀπόκλιμα, for which see Bouché-Leclercq p. 280. In 7:13, if we emend *f'n* to *wa'in*, we can translate ' even if ', and nothing is missing. — **7:18-8:1** *So ... above it*: This makes no sense. Tentatively I suggest emending *wa-lā* in 7:19 to *fa-lā* or *lā*. Then, if Planet A is in the 10th sign of Planet B, I paraphrase as follows: ' on whatever horizon Planet A may be, by necessity it must continue in the 10th sign, for (*fa-inna*) Planet Awill be in its midheaven elevated above Planet B'. — **8:2-5** I am doubtful about the text and translation in line 3, but the meaning seems to be that ' the specialists ' do not accept the mere fact of Planet A being in the 10th sign of Planet B as a reason for saying it is more elevated; they insist that one must take into account the horizon on which this occurs; for if, for instance, Planet A is in the 10th sign of Planet B when the latter is in the 10th house on a given horizon, than Planet A will be in the 7th house and so ' less elevated ' than Planet B. In line 4 instead of ' according to them ' translate ' in the computation of the two ' (relative situations). — **8:11-12** Emend *'l'lwm* to *al-'ulūw* and translate: ' that it is *elevation* in an absolute sense ... except on *the earth*, because the settled regions are in it ' (i.e. in the northern region). People suppose that the north is elevated in some absolute sense, says B., but that is only because the only inhabited regions of the globe happen to be in the northern hemisphere. — **8:18-9:2** and **9:9-10** ' Above ' in all these places is a translation of some form of *isti'lā*, and denotes not physical position but astrological rank. Perhaps ' superior to ' would be a better translation. — **9:2-3** Translate: ' and although to all appearances one can cater for all the other attributes of overpowering by (use of) the word " elevation ", (nevertheless) they used ..... '. By ' overpowering ' (*al-istīlā'*) is meant one planet's influence being stronger than another's. — **9:5** Emend الفرق قرنا to الفوق قربا and translate: ' meaning by the word " above " nearness to the north pole and by " below " distance from it '. — **9:7-9** The theory that a planet ' vanquishes ' a planet which is farther south is found in the *Sūryasiddhānta* VII 21 (Burgess p. 199). We also find in section 23 (Burgess p. 200) the statement ' Venus is generally victor, whether situated to the north or to the south '. This suggests that we should translate in 9:7-8: ' It (Venus) is in the south stronger than *they* (the other planets) are in the north '. But the translation in the text is a legitimate one, and it may be that B. has made a mistake here. Other references to the theory in Sanskrit texts are given by Pingree as follows: A verse of Puliśa cited by Utpala on Varāhamihira's *Bṛhajjātaka* 2,20 and his *Bṛhatsamhitā* 17,10; Viṣṇuśarman quotes *Sūryasiddhānta* VII 23 on *Vidyāmādhavīya* 5,24; an anonymous verse cited by Utpala on *Bṛhajjātaka* 2.20. — **9:7** *bases*: The Arabic is *uṣūl*, which should rather be translated: ' rules '. — **9:11** The ' distance ' here means the difference in *latitude*. — **9:11-13**

For all this compare *Sūryasiddhānta* VII 18-22 (Burgess p. 199). The equation of one degree with one cubit may be connected with the method of observation recommended in that chapter (16-17), by means of two poles each of five cubits stuck in the ground; but the details are very obscure. — **9-14** *testimonies*: This is a translation of the astrological term ἐπιμαρτυρίαι, for which see Bouché-Leclercq p. 165 n. 1. — **10:9-16** B. describes two situations: (1) Sun and moon have the same declination, and the sum of their longitudes is 180°. In the Sūrya-Siddhānta (XI. 2, Burgess p. 273) this is called *vyatīpāta*. (2) Sun and moon have equal but opposite declinations, and the sum of their longitudes is 360°. In the Sūrya-Siddhānta (XI.1) this is called *vaidhṛta*. For a correct explanation of all this see Burgess p. 274. Shukla's account of the passages in the Sūrya-Siddhānta, which is referred to by Kennedy, combines declinations of opposite sign with a sum of 180°; this would contradict B., but seems in fact to be mistaken. Burgess's account also suggests an explanation of the otherwise unintelligible reference in line 14 to the moon's having zero latitude. The Sanskrit text says that (1) takes place when sun and moon are on the same side of the same solstitial point, (2) when they are on opposite sides of the same solstitial point. As Burgess says, this is only true if the moon has zero latitude. B. must have been making the same point, but either through his carelessness, or more probably a lacuna in the text, the reference to the solstitial point has been omitted. — **11:1-2** Translate ' *and* their longitudes ' etc. ' at the equinox ' makes no sense, and the words *iḏi l-i'tidāl* should probably be translated ' when there is symmetry '. For the terms ' ascent ' and ' descent ' see Kennedy [1] and 66:9 ff. in this work. — **11:3-4** Emend *'lmndrh* to *al-munḏira* and translate: ' and that would be one of the signs giving warning of strange occurrences '. — **11:16-17** The meaning is that the planet which rises first of two planets with the same longitude must have a greater northern latitude or a lesser southern one than the other. — **12:2-5** Translate: ' had it not been for *the fact that* it was known ... the sphere of Saturn, then if they said '. — **12:6** The value of 64 earth-radii for the extreme distance of the moon from the earth is derived ultimately from Ptolemy's Canobic Inscription (see Ptolemy, Opera Minora, p. 153,24). Ptolemy calculated it as 64;10 times in the Almagest (V 15, Manitius I p. 311). He makes no calculations about the distance of the planets, but the usual mediaeval method for obtaining planetary distances, which assumed that the maximum distance of each body was exactly equal to the minimum distance of the next more distant body, would, together with Ptolemy's parameters for the planetary orbits, produce a result for the minimum distance of Saturn of the order of magnitude of the 14881 earth-radii given by B. here. al-Farġānī, using that method, gets 14405 (Christmann p. 113). — **12:12-13** That the moon is above the sun is an idea which occurs very frequently in early Sanskrit texts: see e.g. Kirfel pp. 48-49. (Pingree) — **12:19** Perhaps we should read: *mann^an aw kibriyā'^a*: ' as a favour or out of pride '. — **13:13** For the story of Alexander having the *Avesta* translated into Greek cf. Bidez-Cumont II pp. 137-8, where Pahlavi sources are quoted. One of those sources states that one third

of the *Avesta* dealt with astronomy and astrology. Since it is certain that the ' opinion ' of 13:10-11 was not in the Old Persian Avesta, the term ' Avesta ' here (as perhaps always in B.'s works) refers to the collection of doctrine in Pahlavi which was compiled under that name in Sassanian times, and which contained much Greek and Indian material. For the latter see Zaehner, *Dawn and Twilight* p. 195. — **13:18** For the Arabic translation of Philoponus ' work see, in addition, *India* I p. 36, with Sachau's note *ad loc.*, and Steinschneider p. 143. This translation is not known to be extant. However the original is (see my bibliography). The passage B. refers to here is presumably Rabe 318,13 ff. But there all that Philoponus says is that Plato in the *Timaeus* (38 d) put the sun next above the moon instead of in the middle of the planets, so probably B. or Philoponus' Arabic translator misunderstood the passage. — **13:19 ff.** I retranslate the whole passage, though the only corrections in the Arabic text I propose are *an'amū* for *n'mw'* in 13:19, *waṣi'a* for *ws'h* in 14:2, and *al-'uḫar* for *al-āḫir* in 14:3: ' Thereupon *those of them who pondered the matter asked whether it was permissible,* regarding the motions which are found in them (the planets), to put all the planets proper above the sun; *however since* they were (then) left with the space between the two luminaries devoid of a planet to give continuity, *and* (because) it (the space) was large enough to contain the two planets which are isolated *from the rest* by their rotation round the sun . . . (14:6) therefore they considered ' etc. (all one sentence). — **14:10** For the explanation of how the planetary names of the weekdays were derived from the association of the planets with the hours (' the lords of the hours ') see Bouché-Leclercq p. 479 ff. — **14:18** *ascent and descent*: See my note on 11:2. The reference here is to the change in the distance of the planet from the observer. — **15:2** *deferent*: This is the word commonly used in this work to translate *falak al-auj* (literally ' sphere of the apogee '). A better translation would be ' eccentric ', since the eccentric is only a deferent (a circle on which the centre of the epicycle moves) contingently, and in the case of the sun is not a deferent at all. The Arabic for ' deferent ' is *ḥāmil* (used frequently in this work). Usually this mistranslation causes no ambiguity. See however my note on 59:4-5. — **15:8-12** (Note on p. 129). In lines 2 and 4 of this note *B* should be substituted for *Ṭ*. — **15:13-15** For an example of this use of *jawwī* see 90:1. — **15:18-19** Translate: ' due to the discussion of the subject-matter in it *on* chords ', i.e. due to the lay-out of astronomical tables in parallel straight lines. For this derivation of *zīj* from the appearance of the tables see Kennedy [2] p. 123 § 2. In line 19 the text reads *minhū fīhā*: ' from *it* in *them* '. I suggest that this means ' from the epicycle with respect to the chords ', i.e. that the motions in both eccentric and epicycle are set out in tables. — **16:1 ff.** B. attributes the theory that the retrogradations and forward motions of the planets are caused by the tightening and slackening of cords attaching the latter to the sun to ' the ancients ' (cf. 70:4 ff.). Since he has just been talking about Persian astronomers it is conceivable that he means the ancient Persians; and some Pahlavi texts do indeed state that the planets are bound by cords to the chariot of the sun (Zaehner, *Zurvan* pp. 164, 416-417). But as

these texts do not seem to realise the function of these cords, they derive the idea from elsewhere, probably a Hindu source. For the doctrine is found in the *Sūryasiddhānta* (II 2, Burgess p. 53). It is also described by ibn al-Muṯannā in his commentary on al-Khwārizmī's now lost original *zīj*. This commentary is extant only in unpublished (¹) Latin and Hebrew translations. I quote from Bodleian Ms. Arch. Seld. B 34 f. 23ᵛ: ' Multi autem de circulorum natura et situ inscii stellas quasi quodam nexu ligatas bestiarum uel huiusmodi more soli ligatas existimant a quibus dum sol recedit ⟨uel⟩ eaedem a sole separantur hoc nexu sol eas ad sese attrahit '. The doctrine was probably described in al-Khwārizmī's *zīj*, and this is another link with Hindu astronomy. So it may be that B. here had in mind the ancient Hindus rather than the Persians. For more details see Pingree [1] p. 242, from which some of the above is derived. — **16:2-3** The translation is nearly correct, but obscure. The meaning must be: ' their retrogradation (resulting) from the tension of the cord tightened by it (the sun), and *their* forward motion due to its (the cord's) slackening '. This involves changing *ḥrq* to *ḥzq*, and *al-mašdūda bihī* to *al-mašdūd bihā*. — **16:7** *since*: This is not a correct translation of *ba'da an*, and anyway makes no sense. I suggest translating: ' apart from the fact that '. — **16:13-15** This is very obscure. Perhaps translate: ' But if *the meaning* (of *jūyi rāst*) *has to be* " straight table ", then its straightness is the constancy of what is in it *for the meridian* of every locality alike '. This involves changing *lḍ'f* in line 14 to *li-niṣf*, and supposing that B. is referring to the fact that the same arcs of the ecliptic cross the meridian of every horizon in the same space of time. (Cf. *Chronology*, Arabic text p. 6 bottom *wa-kāna ta'dīluhā bi-maṭāli' falak al-burūj 'alâ dā'ira niṣf an-nahār muṭṭaridan fī jamī' al-mawāḍi'*: ' and the equation found from the rising-times of the ecliptic through the meridian-circle is regular and constant everywhere '). These rising-times are what is tabulated in the table of right ascension. He would then be saying that *if jūyi rāst* is interpreted as meaning ' straight table ' (an interpretation which he does not accept), then it means the table of right ascension and is to be explained in the above way. — **16:19** *does not resemble*: Translate: ' does not *differ from* '. This is required both by the sense and by the Arabic. — **17:1** *as to sphericity*: This is the sense required, so in the absence of any evidence that the word in the text (*kryh*) can mean that, we should probably emend it to *kurrawīya*. — **17:5** (Note on p. 130) I know of no grounds for the unlikely statement that the Sanskrit *ucca* is derived from a Greek word. The reference given in the commentary deals only with the derivation of *auj* from *ucca*. — **17:7** *mandūj* is the Sanskrit *mandocca*, meaning the apogee of the eccentric. As B. explains in 17:9-10, the Hindus applied the term ' fast ' (*śīghra*) to the epicycle and ' slow ' (*manda*) to the eccentric. See Sūrya-Siddhānta II 1 (Burgess pp. 53-4) and Neugebauer [1] p. 191 n. 55. This makes the emendation of النظر to البطؤ

(¹) An inadequate text of the Latin translation by Hugo Sanctallensis has now been published by E. Millás Vendrell (Madrid-Barcelona 1963). The Hebrew versions are to be published by B. Goldstein.

in this line certain. Then translate: ' with the addition to it (to *ucca*) of the meaning " *slowness* " '. — **17:15** *Ḥamza ibn al-Ḥasan al-Iṣfahānī*: Far from being unknown apart from the quotation cited in the commentary (p. 130), this individual is well documented, and some of his works (though not those mentioned by B.) survive, and some have even been printed. For details see Brockelmann I p. 152 and Supp. I pp. 221-2, where his dates are given as 893-before 970. B. quotes him frequently in the *Chronology* (p. 61 and elsewhere) and it is clear from these quotations that Ḥamza was particularly knowledgeable on affairs in pre-Islamic Persia, which fits in with the present passage. — **17:18** *bahalā*: Dr. Kripa Shankar Shukla suggests that this is a corruption of the Sanskrit *pātāla*, which means ' hell ', and is used in Sanskrit astrology for ' hypogee '. (Pingree) — **17:19-18:1** *nīḥ* and *nījast*: The reading in the first place should be *nīj*. This represents the Sanskrit *nīca*. This is used to mean the ' depression ' of a planet (as opposed to *ucca*, its exaltation), or to mean ' southern declination '. It is not however used for ' perigee ' (Pingree). So B., as one would expect from 17:17, is merely suggesting a term that the Hindus could have employed. *nījast* is the Sanskrit *nīcastha*, ' standing at depression '. — **18:11** *with it*: Translate: ' *in* it ' (the eccentric circle). — **18:14** The point is that the radius of the earth is so small compared with the distance of any of the planets that the position of the observer can be taken as identical with the centre of the universe. — **18:17** The reference is to Almagest III 3 (Manitius I pp. 162-3). — **19:11** *half it*: Translate: ' the radius '. — **19:12** *equal to it*: i.e. equal to the radius. *adjacent*: this word (*al-mutaqāribaini*) makes no sense. The sense required is ' extreme ', i.e. the greatest and least distances. I dubiously suggest *al-mutafāwitaini*: ' mutually differing '. — **20:3** *for elegance*: Translate: ' because he liked it '. — **20:12** This has been misunderstood; what B. is describing is the angular distance between two points measured from the centre of the circle (D). We can get the correct translation just by omitting most of the words in brackets: ' hence the determination of the (angular distance) between the apogee and (the point of) mean distance '. (Note on p. 133): For $\mathrm{Sin}^{-1}\,\overline{SH}$ read $\mathrm{Sin}^{-1}\,\overline{DS}$. — **21:9** *crux* (*md'r*): Emend to (*miqdār*) (' amount '). — **22:3** Translate: ' when we talk about *the equations* (dual) *of* the two luminaries '. — **22:8** (Note on p. 134): The value of 5;0° for the maximum lunar equation, attributed to Theon in 23:12, is in fact that of the Handy Tables (II pp. 84-5). — **24:9** For maximum lunar equation of 4;56° in Hindu astronomy see e.g. *Khaṇḍakhādyaka* p. 20. — **24:13**: *It resembles getting*: Translate: ' For example they got '. — **24:13-16**: (Note on p. 137): There is a misprint in the rule. It should read: $\beta = \frac{9}{5}\,\mathrm{Sin}_{150'}\,\theta$. This rule is found in the *Khaṇḍakhādyaka* pp. 32 and 82. — **24:19 ff**: (Note on p. 138, 4 lines from bottom): Read: 2;13,20° sin $\bar{\lambda}$. B.'s point is surely that result would have been nearer the common 2;14°, rather than the aberrant 2;13°, as Kennedy supposes. — **25:9 ff**: (Note on *kardaja*, p. 139): It sould be added that within the individual *kardaja* the function increases linearly; it is this that distinguishes it from and ordinary sine function. For Āryabhaṭa on sines see Āryabhaṭiya p. 19. — **26:4 ff**: (Note on p. 141): The reference to *India* should be I pp. xxxv-vi. The text has *wlq* in error for *wyq* at 26:14 as well as at 26:19. The last sentence of the note, even if true, is irrelevant, as it illustrates not division, but *multiplication* by 60. — **27:1-3** The reference is to the *Brāhmasphuṭasiddhānta* 2,2-5. (Pingree) — **27:8** *Nābhāla*: The Arabic is نهبل. This might be emended to *Bahubala* or *Mahābala*, but most probably one should read *Balabhadra*, a well-known early Indian astronomer who is frequently cited by B. in *India*. (Pingree) See especially Sachau's note, II p. 305. He is associated with Āryabhaṭa there too, e.g. I p. 244 — **27:13** *Karaṇasāra*: In *India* I pp. 155-7 B. explains that *Karaṇa* means ' following ' (behind the *Siddhānta*), and *Karaṇasāra* means ' that which has been derived from the *Karāṇa* ', which is correct. (Pingree). Here he explains it as ' Breaker of the Zījes '. The latter translation is applied to a quite different work by Utpala in *India* I p. 157, so it seems that he has made a slip in the present passage. (Note on p. 142): The references to the *Karaṇasāra* in *India* will be found collected only in the Index (II p. 412). Sachau's note is on p. 306 of vol. II. — **27:14** *Vittesvara*: Read ' Vaṭeśvara ', the author of a siddhānta of which a manuscript is preserved in the Library of the University of Lucknow, now published by R. S. Sharma and M. Mishra (New Deli 1962). From this work we learn that his father's name was Mahadatta and his city Ānandapura. Hence correct the note here and al-Bīrūnī, *India* I p. 156. (Pingree) — **27:18** *The Forelock of the Zījes*: In the parallel passage (India I p. 156) Sachau translates the word *ġurra* more accurately as ' blaze '. Dr. Pingree informs me that *tilaka* means among other things a mark applied to the forehead, whence comes the derived meaning ' ornament ', intended by Vijayanandin. (Note on p. 143): Sachau's identification of the work of Abū Muḥammad al-Nā'ib al-Āmulī (called *Kitāb al-ġurra* by B., *Chronology* p. 15) with an Arabic translation of the *Karaṇatilaka* (explained here and elsewhere by B. as meaning *ġurrat az-zījāt*) is based on nothing more than the coincidence of name, and is not in the least strengthened by the present passage. What we know of the contents of al-Āmulī's book from B.'s four references in the *Chronology* (for which see Sachau's note on p. 372) makes it extremely unlikely that this was a translation of a Hindu work. Furthermore, we now know that B. himself translated the *Karaṇatilaka* into Arabic. A manuscript of the translation survives in the library of Pir Muhammad Shah, Dargah, Ahmadabad. See the articles of Nazim and Rizvi cited in the bibliography. I owe these two references to Dr. Pingree. — **28:6-7** *Pulisasiddhānta*: For the clearest statement of the known facts about this work see Pingree [1] p. 237 n. 63. It is clear that there are no good grounds for the identification of the author of any of the works going by the above name with Paulus Alexandrinus, the author of the Greek astrological work εἰσαγωγικά, though B. himself did so. (Note on p. 143): ' measured in degrees of arc along the deferent ': i.e. the circumference of the epicycle was expressed in 360ths of the length of the circumference of the deferent. The parameters of the *Khaṇḍakhādyaka* referred to in the note are themselves derived from the Midnight (*Ārdharātrika*) system of Āryabhaṭa I.

(Pingree). — **28:8-10** The parameter 31;26° for the lunar epicycle is an error of B. (not the copyist, as is clear from Kennedy's note on this passage) for 31;36°, as is shown by the following note of Pingree: In the *Brāhmasphuṭasiddhānta* (2,20) the epicycles are pulsating; the solar epicycle varies between 13;20° and 14°, the lunar between 30;44° and 32;28°. The mean values are 13;40° and 31;36° (compare B.'s 13;40° and 31;26° here). Brahmagupta's parameters for the *Brāhmasphuṭasiddhānta* are taken from the *Paitāmasiddhānta* of the *Viṣṇudharmottarapurāṇa*; and on p. 4 of the latter, which does not have pulsating epicycles, one finds the values 13;40° and 31;36°. — **28:10** Emend ادا رَوا to اداروا and translate: 'that they *made revolve*, round the centre of the deferent, and with a *radius* of the sine ... a circle'. I presume that 'round the centre of the deferent' means 'with its centre on the deferent'. — **28:14-18** (Note on p. 144): The reference to al-Khwārizmī's *Algebra* should be Rosen pp. 198-9. In that place references to this approximation to π in Sanskrit literature will be found. — **29:14-18** (Note on p. 147). It appears from the curious wording of this note that the Handy Tables were not available to the writer. Let me therefore confirm his conjecture that cols. 3 and 4 of the Almagest table are combined into a single column in the Handy Tables, and that the latter are numerically essentially identical with the former. It should be noted however that both in the Almagest (XI 11, Manitius II p. 264) and in the Handy Tables (II pp. 174-5) the maximum equation for Venus is 2;24°, so B.'s 2;23° is an error (correct Venus' value in the notes on 30:4-9 and 31:9-31:14). — **30:4-9** (Note on pp. 147-8): Regarding the list of ibn al-A'lam's values for planetary equations, note that at 23:2 B. gives this author's maximum solar equation as 2;0,10° (i.e. 0;22,50° less than the Almagest). It is this fact *combined with* the statement about Venus in 30:1 which makes it probable that it is the *Almagest* values with which B. is comparing ibn al-A'lam's. — **31:17-32:2** (Note on p. 150): Read 'except for *Jupiter*' instead of 'except for Venus', and *32*:14 instead of 28:14. — **32:6** We should read 5;*50*, with the Arabic text. The first formula on p. 151 gives 5;50, and not as Kennedy states 5;10, so there is no basis for his emendation. Furthermore 5;50°, though low, is credible as the maximum deferent equation of Saturn (while 5;10° is not). — It is quite near the 5;43° attributed to ibn al-A'lam on p. 147. — **32:14** Here for the maximum equation of Mars we should of course read 11;[8],30. — **35:2-3** The 'three equal lines' are HS, SD, DṬ. — **36:6-7** Read, with the Arabic text: 'since point Ṣ, which was at A'. The point of what B. is saying here is as follows: if you imagine the point Ṭ, the centre of the deferent, moving round D and carrying the deferent with it, then when Ṭ reaches the position S, A, which was the apogee, will have reached point Ṣ, and Z, which was the perigee, will have reached point W. But Ṣ is now nearer to the earth H than W; thus 'apogee' is nearer than 'perigee', and so the two have 'changed places'. — **38:8** *ABJD*: The points Ḥ and J have become interchanged in Figure 4 both in the Arabic and the translation. The translators have altered the text to agree with the figure from 38:17 onwards, though not here and in 38:9, where there is a discrepancy. They should have altered the figure to

agree with the text, but except in the above two places no ambiguity arises. — **41:2-43:15** (Note on p. 155): The maximum equation is *not* arc AB. It is just this false identification which B. says (43:9-14) is the cause of Abū Ma'šar's erroneous formula. (p. 156): In the first line read: $2\overline{ZA}.\frac{1}{2}\overline{SA} = \overline{AM}.\overline{AJ}$. Abū Ma'šar's rule (last line but one) should read: $90^\circ + \mathrm{Sin}_r^{-1}\frac{(\mathrm{Sin}_R r)^2}{2R}$. — **42:4** *the epicycle equals*: Translate: 'the epicycle *is like*'. — **43:9** Translate: 'he means *by* the epicycle radius the arc AB', i.e. B. claims that Abū Ma'šar was saying that the maximum equation
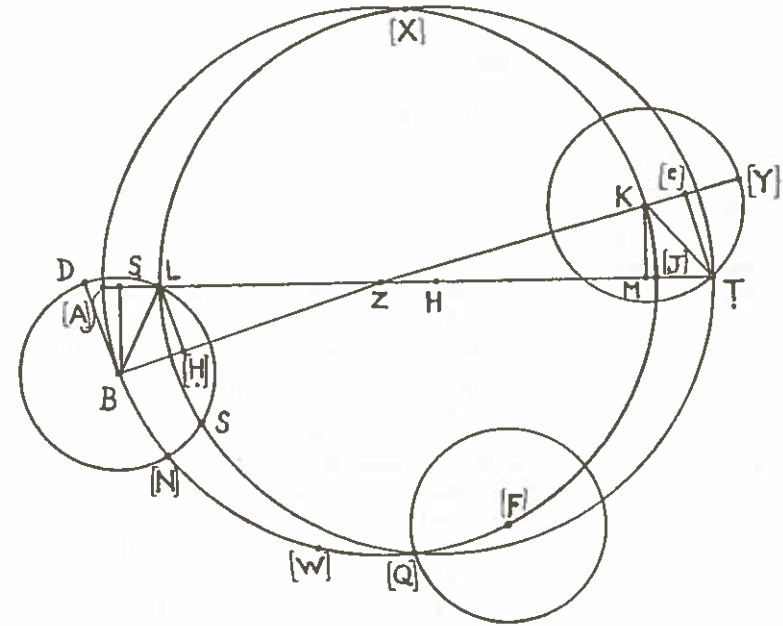
Fig. II

(which is given by the arcSine of the epicycle radius) is arc AB. We may doubt whether B.'s interpretation of the extraordinary expression 'the sine of the epicycle radius' is correct. I would rather suspect a corruption in B.'s text of Abū Ma'šar. If for the above expression we substitute 'the epicycle radius', Abū Ma'šar's rule becomes the same as B.'s. — **43:15** *was known to him*: Translate: 'is a fixed quantity in relation to it' (arc AṬ). — **45:4** Emend *w-ldwyr' 'wlh 'w* to *au tadwīran ūlā au* (Perhaps instead of اولى the vulgar form اوّلة was used). Translate: 'the first or uncorrected argument or epicycle' (cf. 46:2). — **46:2** Translate: 'the anomaly, or the corrected argument or epicycle'. — **47:13 ff.** The following section has been rendered unintelligible in places by the mistaken emendation of

Figure 8 and misinterpretation of the text. I have therefore redrawn the figure. See my Figure II, which reproduces the figure in the Arabic text except for a few emendations and additions of letters. This correct figure should be referred to with the following notes. — **47:15** *the other two*: i.e. the right and left mean distances when the epicycle centre is at apogee (the two positions of the ' mean depression ', which are ' a little bit elevated above ' the depression at L). — **48:3-7** To achieve English syntax translate: ' And because BZ is the radius ' ... (line 5) ' and BL is the radius of the epicycle, triangle BLZ... will be known ' ... (line 7) of its base. And BḤ is the sine of arc DL, so if ' etc. — **48:14** Read ' S[N] ' to avoid ambiguity. Clearly W is distant from Q by the same amount as F (i.e. by the radius of the epicycle), but on the opposite side. — **48:15** Translate: ' will be at the *intersection* ' (i.e. at Q). — **49:8-50:1** Translate: ' and its (arc TY's) complement is the total elevation, and its (the elevation's) beginning is from when the centre is at [F], since the right mean distance ' etc. If the text is right, B. is slipshod in saying that the complement of arc TY *is* the total elevation, as comparison with the parallel situation in 48:9-10 shows that forming (90°-$\widehat{TY}$) is only the next step towards finding the ' total elevation ', which is the arc of the epicycle cut off between the two circles when the centre is at K. The second part of the above sentence means that this arc ceases to be called ' depression ' and begins to be called ' elevation ' when the centre has passed F (and so the right-hand mean distance has passed Q). — **50:2-3** The ' point of intersection ' is Q. It is clearly correct to say that the arc between Q and F is given by the formula $2\,\mathrm{Sin}_R^{-1}\left(\frac{r}{2}\right)$, where R and r are the radius of the deferent and epicycle respectively. — **50:8** Emend *yšbh* to *bi-šibh* and translate: ' And in imitation of the operations ..., we transfer '. — **50:11 ff.** *the beginnings of the distance sectors*: These are the sectors of the deferent (see Kennedy's note on p. 128) and are the points A, Q, J, X in my figure. — **50:16 ff.** *the arc of the chord*: Cf. 50:3. What B. is saying is that the epicycle centre is to the right or left of X by an arc equal to $\widehat{FQ}$ when the epicycle circumference passes through X, etc. — **51:1** To understand this passage compare 50:4. For instance ' the distance ' in 51:1 is equal to ṢB. — **51:8** *altitude*: Translate: ' elevation '. — **51:12** *the center*: i.e. the centre of the epicycle. This word (*markaz*) is the technical term for ' mean anomaly ', which is really what is meant here. — **51:13-14** Translate: ' the distance *of* the centre *from* the *designated* beginning ', i.e. the beginning of the elevation or the depression as the case may be — **47:13-51:18** (Note on pp. 158-9): The obscurity which the commentator finds in this passage is due to a misunderstanding, which the above notes should have cleared up. Apart from the inevitable corruption of the letters designating points in the diagram, the text appears sound. As for the point of the passage as a whole, that is made clear in 51:16-17: it is to find the position of mean distance on the epicycle for a given position of the epicycle on the deferent. It is therefore a natural outcome of the discussion (38:3 ff.) of the division of the epicycle into distance sectors; and the following sections on the parameters are necessary in order to carry out

the calculations and tabulations outlined in this section. The last five lines of p. 158 and the first six of p. 159 should be deleted, as they are based on a total misunderstanding of the text at this point. The commentator is correct in his supposition that the reference in 50:8 ff. is to the normal interpolation procedure in planetary tables for positions of the planet between mean and extreme distance. Moreover the rule given in 51:13 ff. is an example of a similar interpolation procedure. To obtain a correct statement of the rule, alter the commentary on p. 158 to read: ' where δ is the deferent arc from the position of *zero elevation or depression* to the position of the epicycle centre'. — **52:5-7** These parameters are indeed taken from the Almagest, but they are in each case the first approximation arrived at by Ptolemy, under a preliminary hypothesis (for which see *Almagest* X 7, Manitius II pp. 178-9). They are then corrected, and Ptolemy's final parameters are those quoted in the commentary (p. 159). There seems no explanation except carelessness for B's use of these approximate parameters. The references to the Almagest are: Saturn: XI 5 (Manitius II p. 233). Jupiter: XI 1 (Manitius II p. 208); here twice the parameter is given as 5;23 and on p. 210 half of the latter is rounded to 2;42. Mars: X 7 (Manitius II p. 184). For Venus and Mercury the parameters given will be found at *Almagest* X 3 (Manitius II p. 163) and IX 9 (Manitius II p. 144) respectively. — **52:15** *without* HṬ: Translate: ' *and not* HṬ '. — **52:19** Translate: ' and *it* (ḤD) *is less than* it (DK) '. — **53:6** As the commentary states, the figure for Mercury should read ' twenty-two parts and a half '. However, no ' restoration ' is necessary, as it is already in the Arabic text. The references to the Almagest for the parameters of the epicycle radius are as follows: Saturn XI 6 (Manitius II p. 246). Jupiter XI 2 (Manitius II p. 223). Mars X 8 (Manitius II p. 198). Venus X 3 (Manitius II p. 163). Mercury IX 9 (Manitius II p. 144). — **53:8** Translate: ' have followed in *them* ' (the maximum equations). — **53:6-55:13** In the comparative table given on p. 160, the following points scould be noted: (1) The figures given for *Theon's Canon* are identical with those of the Handy Tables except for Mars, where Halma's text (II pp. 166-7) has 41;*10*. The agreement of B.'s figure with al-Battānī's indicates a split in the manuscript tradition here. (The oldest ms. of the Handy Tables, Vat. Gr. 1291, also has 41;*9* in this place). (2) The figures for Jupiter and Mars given for ibn al-A'lam are not explicitly stated by B., but merely inferred from his silence. (3) The figure for Saturn in the Shāh Zīj should read —0;0,8. It seems most unlikely that this is correct. — **54:8-14** *forty-one parts and thirty minutes*: This is either a copyist's error or a mistake by B. for ' forty parts and thirty-one minutes '. It is stated (54:13-14) that the parameters in the Shāh Zīj are the same as those in al-Khwārizmī. I have listed the latter in my note to the table on p. 100 (see Table II, p. 55 of this article). And the parameters of the Shāh Zīj were derived by Kennedy from a later passage and are listed on p. 178 of the commentary (under *Māshāllāh*). Both agree in the parameter 40;31° for Mars' maximum equation of anomaly. In all other cases both agree with the figures given by B. here and listed on p. 160 of the commentary (though in some other values there

is a discrepancy of one minute between al-Khwārizmī and the Shāh Zij). On the other hand Abū Ma'šar's parameter in the list on p. 178 is 40;30°, an example of the 'one minute less' of 54:9. The parameters of Abū Ma'šar listed on p. 178 are very similar to those of the Shāh Zij, and are inconsistent with the two given in this passage (54:9 and 54:13). But it would not be surprising if Abū Ma'šar used different parameters in different parts of his work. — **55:1-11** Āmarāja on *Khaṇḍakhādyaka* 2,13-14 quotes the following as the circumferences of epicycles of the conjunctions of the planets from Pauliśa: Saturn 39;30° – Jupiter 72° – Mars 233° – Venus 260° – Mercury 132°. Except in the cases of Saturn and Mars, these parameters are from the Midnight System of Āryabhaṭa I. The verse of Pauliśa cited by Āmarāja also gives the rule for computing the maximum equation: $\text{Sin } e_{max} = \dfrac{\text{circumference of epicycle} \times R}{360}$. Using $R = 150$ (the value of the *Khaṇḍakhadyaka*) rather than $R = 3438$, Āmarāja then computes the maximum equations according to Pauliśa's rule, and gets: Saturn 6;20° – Jupiter 11:30° – Mars 40;30° – Venus 46;15° – Mercury 21;30°. (Pingree) Both of the above sets of figures should be compared with those given in the table on p. 160 of the commentary. The discrepancies may incline us to believe that B. cannot be referring to the same work as Āmarāja. Moreover the conversion rule given 54:19-55:2 (p. 161 of the commentary) is, unlike that given above, only an approximation, which leads to quite large errors when the epicycle is large. More than one *Pauliśasiddhānta* existed: see Pingree [1] p. 237 n. 63. — **56:9** *external*: Translate: 'eccentric' (*al-ḫārij*). The technical term for 'eccentric circle' is '*falak ḫārij al-markaz*'. — **56:17** Translate: 'issuing from Z (and) H *to them*' (to A and J). The next sentence makes no sense. Perhaps the best explanation is that at these two points, A and J, the disparity in quantity (*tafāḍul*) of the two *lines* is at its greatest. This would necessitate reading *tafāḍulhumā*. — **57:4-5** *they are equal to circle ABJD* etc.: This is patently false: each of the circles in question is equal to a quarter ABJD, as the diameter of each is equal to the radius of ABJD. Moreover mention of ABJD is irrelevant: the point is that the circles are equal to each other. The error is too gross to attribute to B. himself. The simplest solution is to delete the words 'to circle ABJD ... one magnitude' as an ignorant interpolation. — **58:2** *ZMB*: Emend to 'ZMS' or to 'ZMH'. — **58:11-59:9** Figure 11 of the text is wrongly drawn. This mistake has led the translators astray and makes the argument difficult to follow. I have therefore redrawn it (see my Fig. III). Ḥ is a point on the deferent (on the extension of the tangent ZD), and not a point on the epicycle. It is then easily shown that ḤṬ equals the radius of the epicycle. — **58:14-15** Change the punctuation to read: 'the maximum equation, because the lines extending' (from Z). — **58:16** Read: 'angle BZḤ' with the Arabic text. — **58:17** *any angle bounding it*: What is needed is 'any angle *bounded by them*' (the lines of 58:15). The Arabic is تحيط بها. The simplest reading is تُحِيطُ بِهِما. — **58:19-59:1** Translate: 'because both it (ḤṬ) and BD are perpendiculars from one end of the arc (BḤ) to the

hypotenuse drawn from its other end', i.e. $ḤṬ = BD$, because each is the perpendicular of an equal right-angled triangle. — **59:4-5** This is correct, for $\frac{SB}{DB} = \frac{DS}{DZ} = \frac{ḤT}{ḤZ}$, and ḤṬ, as shown above, equals the radius of the epicycle, while ḤZ is equal to the radius of the eccentric. Here the rendering of فلك الاوج as 'deferent' (see my note on 15:1) is unfortunate, for what B. is saying is that SB:DB equals the ratio of the eccentricity to the radius of the eccentric *when one substitutes* the eccentric model for

Fig. III

the epicyclic. The eccentricity is thus nothing to do with the distance ZH in Figure 11, as the translation would lead us to suppose, but instead is equal to the radius of the epicycle. — **60:3** Translate: 'lower than it, *or due to* their being above the mean distance' etc. In 60:1-3 B. is suggesting two alternative explanations of why the first and fourth sectors are called 'ascending' and the other two 'descending': (1) Because the centres of both epicycle and eccentric circle are 'raised' (صاعد) from the centre of the universe; so the two sectors which are in the same direction are said to be 'ascending' as well. (2) In the first and fourth sectors the distance to the body is greater than ('above') the mean,

while in the other two it is ' below ' it. — **60:18** *equal to*: Translate: ' like '. — **61:1-3** Text and translation are doubtful but the meaning is clear: at the righthand stationary point on the epicycle the planet begins its (apparent) forward motion, and accelerates until it reaches its maximum speed at the epicyclic apogee, where it begins to decrease speed until it reaches the left-hand stationary point, after which it becomes retrograde (as explained in 61:4 ff.). — **61:6-7** Omit ' (the end of) '. ' one ' and ' the other ' refer to the two stationary points. — **61:13-14**
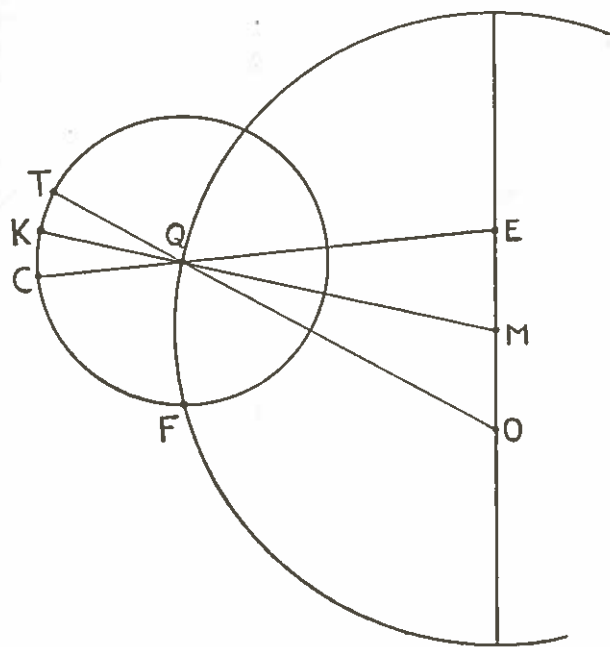
Fig. IV

Translate: ' should be given precedence ...*over* the change in the equation ... unless *an* effect is claimed *in* it '. — **61:18** *the first method rather than the second*: i.e. distance rather than velocity sectors. — **63:12** *by them*: Translate: ' for them ' (for the sectors). — **63:14-17** (Note on p. 165): For ' 270° ' read ' 540° '. — **63:18-64:6** (Note on p. 165): The explanation given in the commentary of the reason for the use of half the equation of centre is wrong. See my Fig. IV. The (mean) anomaly is the arc CF. According to the ' first opinion ' (64:1) this is adjusted by ' half the equation of centre ', i.e. by $\frac{1}{2}$ (arc TC = $O\hat{Q}E$). This will make the first sector approximate to the arc KF (only approximate, as B. indicates in 63:19, because $\widehat{TC}$ is only exactly twice $\widehat{KC}$ when $E\hat{M}K$ is a right angle). Thus the first sector is measured from the point on the epicycle which is ' apogee ' with respect to M, the centre of the deferent (this is what is

stated in 64:2-3). According to the ' second opinion ' (64:5) the anomaly is adjusted by the whole equation of centre, and the first sector becomes arc TF, i.e. it is measured from the true apogee of the epicycle. In 64:5 translate: ' the anomaly is considered to be adjusted by the whole equation of centre '. — **65:5** *motion of the difference*: i.e. simple anomalistic motion. — **65:12 ff.** B. suggests that by considering the four distances in the epicycle at each of the four points in the deferent one would get $4 \times 4 = 16$ different distances. But he immediately discounts this on the grounds that the right-hand mean distance does not differ (in length) from the left-hand mean distance, so one has only $3 \times 3 = 9$ different distances. — **65:15** Translate: ' it would not become sixteen by repetition of the rotation, whether ' etc. — **69:5** Translate: ' what characterizes the motion of the moon *with respect to* the deferent *is little* (*yasīrun*) compared with the motion of the centre ' (of the epicycle), i.e. when the moon is in the upper half of the epicycle its motion on the epicycle, as seen from the earth, is less than the motion of the centre of the epicycle, though in the opposite sense. Hence there is only slowing down (69:7), no retrogradation. The word translated ' deferent ' is *al-ḥawāmil*. The only explanation I have for the use of the plural is that the moon's deferent is not fixed in space, but has a variable centre (see 33:10-12), i.e. the moon has many possible ' deferents '. — **69:10-12** Translate: ' the motion of the planet *with respect to* the deferent ... does not differ from the moon's ... in slowing down and deceleration ' (emending *'ltsbyṭ* in 69:12 to *at-taṭbīṭ*), i.e. the planet, in that part of its epicycle where its motion is contrary to, but still less than, the motion of the centre, is like the moon in the upper part of its epicycle in that it slows down. — **70:18** It is clearly necessary to emend ' second ' to ' third ' (*aṯ-ṯānī* to *aṯ-ṯāliṯ*). Cf. 68:2-6. The note on p. 168 (third paragraph) only makes sense with this emendation, so its omission is just an oversight. — **71:4 ff.** Translate: ' *increasing* and *decreasing* in the two even ones. ' ' in both of its directions ' (or ' *regions* ') may mean ' both when northerly and when southerly '. The latitude is increasing (i.e. the planet is travelling northwards) in the first and fourth quadrants, though in the first the planet has a northerly latitude, in the fourth a southerly. ' the remaining ones ' (line 5) are of course quadrants, *not* sectors. — **71:6-7** Translate: ' the first quadrant *which* is ... and the third quadrant... (*are*) '. — **72:10-11** *becomes suspended* (*ta'allaqa*): This means ' is conditional ' rather than ' becomes indeterminate '. The literal meaning of the next sentence is: ' and perhaps it is as if the planet in both of them is the descent '. This is plain nonsense, and the mistranslation in the English text does not really make sense either. It seems probable that there is a lacuna after ' both of them ' in 72:11. In this lacuna came the word ' ascent ' (*aṣ-ṣu'ūd*). Then it is ' the ascent and descent ' which are ' dependent upon the unmodified center and the true anomaly '; and ' both of them ' in 72:11 and 72:12 ff. refers to the two deferents which are discussed in 72:3 ff. The introduction by the translators of the idea that ' both of them ' refers to ' the deferent and the epicycle ' leads to impenetrable confusion. — **72:14-15** Translate: ' and they are of two kinds, one *relative to* the mean distance '.

— **73:4-7** This passage is perfectly intelligible (whereas the 'emendation' suggested on p. 171 would indeed be baffling). The commentator's difficulty comes from a failure to realise that the mean 'longitude' of an inner planet is defined in Hindu astronomy in a different way from that in which Ptolemy defined it (which was the mean longitude of the sun). Strictly speaking, it is not a 'longitude' at all; it is known technically as the 'mean conjunction'. The following account is substantially con-
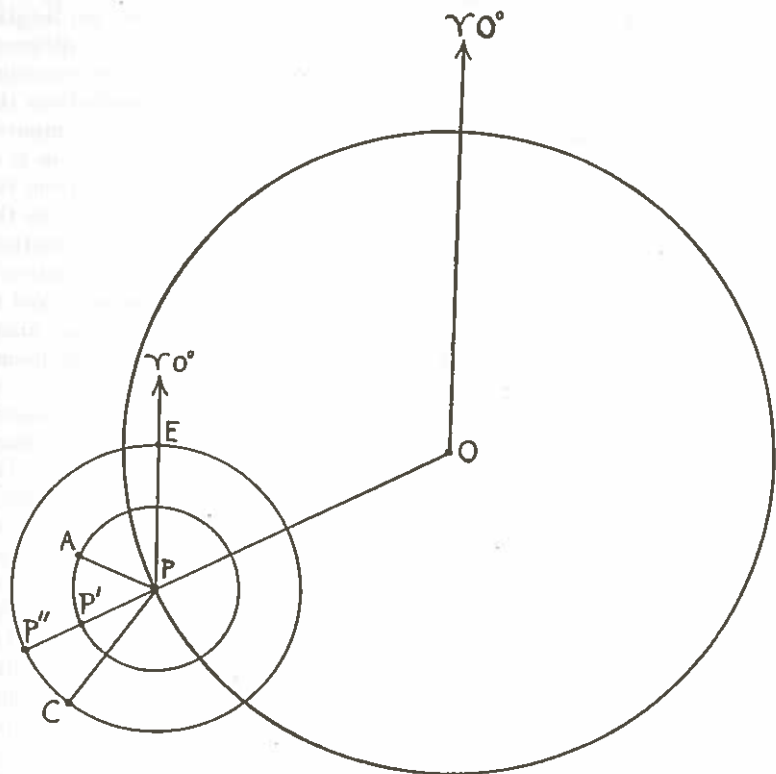
Fig. V

tributed by Dr. Pingree (See Fig. V): "The mean planet P (which is the mean sun for the inferior planets) travels on a circle concentric with the earth O, and about the mean planet rotate two epicycles, one of the apogee (AP'), the other of the conjunction (EP''C), which allow one to compute the inequalities, but in no sense represent a kinematic model of the motion of the true planet. The line from the centre of the earth to the mean planet is extended to meet the circumferences of the two epicycles, and the longitudinal distances of the planet's apogee and its 'conjunction' are marked off from Aries 0° of the epicycle (these are located at A and C respectively in Fig. V). Then for a superior planet the conjunction is the

mean sun; for an inferior planet it is the sum of the motions of the mean sun and the anomaly of the Ptolemaic system (EC = EP'' + P''C); and in computations, for both superior and inferior planets one subtracts the mean planet from the conjunction. (For an account of the subsequent procedure for finding the true longitude see Neugebauer [1] pp. 174-182). It was in fact the desire to make the procedure for computing the position of an inferior planet exactly the same as that for a superior planet which led to the above definition of 'conjunction'". Thus the 'mean conjunction' is tabulated in Hindu mean motion tables (cf. e.g. *India* II p. 16), and it is this that B. means by 'mean of the planet' in 73:6. He is clearly correct in his statement that the difference between this (EC) and the mean longitude of the sun (EP'') gives the mean anomaly. This 'mean conjunction' is of course equivalent to 'sidereal' mean motion, but the above explanation shows that it is quite wrong to impute a heliocentric theory to the Hindus on the strength of this, as has been done. B. was not alone in calling this 'mean motion'. Exactly the same statement as 73:6-7 is found in a fragment of al-Khwārizmī's original *zīj* preserved in ibn al-Muṯannā's commentary (see my note on 16:1 ff.), Bodl. Ms. Arch. Seld. B 34 f. 20ᵛ: 'medium stellae cursum in stellis superioribus rectificans de medio cursu solis abstrahens primam porcionem (= mean anomaly) relictam inuenies. nam pro inferioribus medium cursum solis utriusuis eorum medio cursui detrahere mandat'. — **73:7 ff.** Translate: 'And if it (the mean anomaly) is substituted for the difference, mentioned for the superior planets, between their mean and true longitudes the result deviates from its original value (?). And if ascent and descent in the epicycle are determined through it, then, if' etc. B. is making the obvious point that determining the sectors by using the mean anomaly will give a different result from that obtained by using the equation of anomaly (the difference between true and mean longitudes) because the same anomaly will produce different equations according to the position of the epicycle on the deferent, as he goes on to explain. — **74:17** Read: 'angle ZK[H]'. This is shown to be necessary by the next sentence. — **75:1** *is inaccurate*: Rather 'is invalid' (*yabṭulu*). B. is saying that this kind of representation of the combined effect of the two equations is improper. — **75:15** The last two words in the line, *'lty m'n'*, are clearly corrupt, but if we omit them the sense is plain: 'and angle ZṢH falls short of equality with the base'. — **75:18** Translate: 'if only he had'. The sentence has no apodosis (see next note), but is a wish. — **75:18-76:8** The following is a rough explanation of what I think B.'s point is here: In Figure 13 of the text, if the planet is at B and the centre of the epicycle at R we have addition of the two effects (epicyclic equation and equation of centre); if the planet is at B and the centre at D we have subtraction of one from the other. In both cases each equation is near the maximum, and in both cases the planet is at mean distance. So the rule of Abū Ma'šar (74:8) is disproved. The 'arc of half the eccentricity' (75:18) is angle TBZ. This added to ninety degrees gives angle AZB, and the equation of centre at that point (translate '*at* that amount' in 75:19) is computed. Cf. 20:12-15 with my note. 'twice the arc of one fourth the

diameter of the epicycle (75:19-20) is angle RZB. The latter added to the above equation of centre will give the difference between mean and true longitude of the planet ($A\hat{Z}R - A\hat{H}B$) *approximately* (76:1) — not exactly, because for simplicity B. assumes the equation of centre at R equal to that at B, whereas in fact it will vary somewhat from one to the other, depending on the size of the epicycle. As B. says (76:1-2), these two equations are in each case not much short of the maximum. At 76:2 emend '*m*' to '*alaman* and *d* to *r* and translate: ' and made *it* (the sum of the two equations) an indication for the mean distance at R' (cf. 76:7). At 76:3 ff. translate: ' so that the sum of the two equations should be a measure for it (the mean distance) when the increase or the decrease of the two is together ', i.e. when both epicycle and eccentric equations have the same sign, the fact that their sum is equal to that determined above will be an indication that the planet is at mean distance. Similarly 76:5-8 states that the amount of the difference between the two equations when they are of different sign is an indication whether the planet is at mean distance. Text and translation again both need emending. The sense is: ' And we take ' (reading *wa-na'ḫuḏu* for *wa-ya'ḫuḏu*) ' the excess of twice the arc of one *quarter* of (the diameter of) the epicycle... and make it an indication for the mean distance at D and its opposite point in the fourth *quadrant*, so that the difference ... should be a measure for it '. In 76:5 ' the arc of half the eccentricity ', if the text is not corrupt, is a careless abbreviation for ' the equation at 90° plus the arc-sine of half the eccentricity ' (cf. 75:18-19). — **77:2** Translate: ' except if we consider '. The ' two orbits ' are the spheres of epicycle and apogee. — **77:4-5** Translate: ' he is *more entitled* to omit it at this place '. — **77:10-78:5** Abū Ja'far said that the increase or decrease in the distance sun-earth is in inverse proportion to the equation, and that mean distance is at maximum equation (77:10-14). B. objects that even if this were so (which it is not, 78:2), it would give no criterion for deciding the sectors, i.e. whether the distance is increasing or decreasing. — **77:13** Translate: ' the ratio of the equation to its maximum is equal to the ratio of the chord which belongs to that equation to *its* maximum '. — **79:8** *hastens* Read *fa-insamā* for *f'nšmr* and translate: ' is elevated '. — **80:1** *extended*: This makes no sense. Probably *mdwd'* should be emended to *mardūdan* (' reflected '), though what the difference is between a ray being ' reflected ' and being ' sent ' remains obscure. — **80:4-5** ' depression ' and ' elevation ' here mean in effect ' decrease ' and ' increase '. — **81:16** *true longitude*: This should be translated ' denominator ', as is clear from Kennedy's note on the passage (p. 173 bottom). The statement in that note that B. is unable to explain Abū Ma'šar's reasons for doubling the denominator is not true: the whole of 81:19-82:16 is devoted to an explanation (all that B. says in 81:18 is that the explanation may not be *correct*). B. supposes that because the sun and moon, unlike the other planets, have no equant, Abū Ma'šar therefore made the distance between the centre of the *deferent* and the centre of the universe for sun and moon correspond to the distance between the centre of the *equant* and the centre of the universe for the other planets. So he doubled the ratio corresponding to the distance

between centre of deferent and centre of universe for the planets in order to get that for the sun and moon. Consequently the final ratio for the latter came out to twice the normal $\frac{4}{25}$. — **83:15-84:4** You have x minutes: multiply by 48 and divide by 5; this leaves a remainder. Multiply the remainder by 12: this gives you a sexagesimal number of the form 0;0,b,c (where b are seconds and c thirds of a degree) which is equal to $\frac{4}{25}x$. ($\frac{4}{5}$ = 0;48, 84:1). In 84:3 B.'s point is that al-Farġānī multiplied by $2 \times 6 = 12$, which is equivalent sexagesimally to dividing by 5). This explains why the division by 5 in 83:16 is not ' redundant ', as is stated on p. 174 of the commentary. — **85:1** i.e. $\frac{7}{22} \approx 2 \cdot \frac{4}{25}$. — **85:9** (Note on p. 175): 0;96,36 is a misprint for 0;9,36. — **85:12** *so far as these numbers are concerned*: Translate: ' in that number ' (namely 60), i.e. 60 is not to be explained as the maximum size of the transit, but the table is a multiplication table (85:14-17). — **86:7 ff.** This section, as explained by Kennedy's note on p. 176, is quite straightforward, but has been complicated by mistranslations. The following corrections should be made: **86:7-8** ' And they (i.e. the minutes of transit), in the first sector as determined by the equations, are the magnitude ' etc. The above translation of *al-maqsūm bi t-ta'ādīl* is dubious (I take it to mean ' distance sector ' as opposed to ' velocity sector ', and ' equations ' (plural) is used because both apogee and epicycle sectors are meant). It may be preferable to omit the words as corrupt; but in any case both grammar and sense forbid application of them to ' the minutes '. — **86:9** Translate: ' So if the minutes of *transit* '. **86:10 and 13** Substitute ' we subtract ' for ' he subtracted '. **Figure 14 (p. 97)**: The figures printed in this figure are substantially those of the rather corrupt Arabic text. For convenience I tabulate here the emended figures suggested on pp. 177-8 of the commentary, adding my own emendation for the radius chord of Saturn. The latter is the result of multiplying 0;9,36 by 5;43 (one minute less than Māšāllāh's figure; cf. the parameters for the maximum epicycle equation of Mars in the comparative table on p. 178). The emended digits are in italics:

Table I

Sun	Moon	Saturn		Jupiter	
Apogee Chord	Radius Chord	Apogee Chord	Radius Chord	Apogee Chord	Radius Chord
0;42,*52*,*48*	1;34,43,*72*	1;22,43,*72*	0;*54*,*52*,48	0;48,*57*,36	1;4*3*,19,*72*

Mars		Venus		Mercury	
Apogee Chord	Radius Chord	Apogee Chord	Radius Chord	Apogee Chord	Radius Chord
1;47,12,0	6;2*8*,48,0	0;42,*52*,*48*	7;32,*57*,36	0;38,24,0	3;26;24,0

88:15 *a reconciler by*: Translate: ' many-branched in '. — **88:18** ' the equality ' is presumably not ' of the equations ', but that referred to in 88:10, i.e. equal distance of the two planets from their respective apogees. — **89:4** ' *magnitude* ': The Arabic is *jirm*, ' body ', as in 88:11 and 89:2, and clearly here too the meaning is ' apparent diameter '. In 89:5 translate: ' half the sum of the two bodies '. What B. means is that in order to determine when two planets come into conjunction, in the sense of just touching, one must take half the sum of their apparent diameters: the latter will be the difference between their longitudes at the configuration in question. — **Figure 15 (Table on p. 100)**: In the top row of this, for the relation between Venus and Saturn read 2962 [7]. The statement in the commentary (top of p. 180), that all the ratios involving the deferent equation of Mars are mutually inconsistent, is correct, but one can still determine limits within which it must lie: these are 11;10° and 11;13°, so it cannot differ much from Abū Ma'šar's value (p. 178) of 11;10°. The values of al-Khwārizmī are very close: for convenience I append a table of them:

TABLE II

		Degrees
Sun		2;14
Moon		4;56
Saturn	Centre	8;36
	Anomaly	5;44
Jupiter	Centre	5;6
	Anomaly	10;52
Mars	Centre	11;13
	Anomaly	40;31
Venus	Centre	2;14
	Anomaly	47;11
Mercury	Centre	4;2
	Anomaly	21;30

90:13 *the mentioned opinions*: For these see the commentary p. 169 ff. — **91:4** *the composite ratio*: This is $\frac{4}{25}$ etc. See 80:8 ff. — **92:9** *computed by the sines*: In the less sophisticated Hindu astronomical works the maximum equation of centre is assumed to occur at a mean longitude of 90° (cf. 92:1) and between 0° and 90° is directly proportional to the sine. See, besides the references given on p. 180 of the commentary, Toomer [1] p. 12. — **92:12** *from which*: Translate: ' for which '. — **93:1-2** Translate: ' So if the maximum equations *were* equal for both planets, and furthermore its (the maximum equation's) position *were* at the epicyclic apogee, *then* the *portion* of planet Z would be more and the *portion* of planet Y would be less '. The ' portion ' means the amount of the maximum equation appertaining to the planet at that point. B. is saying that 'Umar's ' law of elevation ' (93:3 and 10 ff., where the law is formulated, cf. 89:17-90:8) would only produce something like the right ' elevation ' according to the ' first ' and ' second ' opinions if the maximum (epicyclic) equation occurred at the apogee of the epicycle, which is the exact opposite of the true situation (93:4). — **93:10** *at it*: Translate: ' according to it ' (the law). — **92:12-93:17** (Note on p. 180): The commentary is correct in stating that *before* comparing elevations between planets, one of the elevations should be multiplied by the proper coefficient. Note however that in the example at 94:18-19 the coefficient is not used until *after* the comparison has been carried out. — **94:5-6** Emend *'ḥd* to *aḫḏ* in line 6, and translate: ' and we make the required condition for ascent the taking of the difference (i.e. the supplement), with agreement of the sign at their base ', i.e. we allow ourselves to take the supplement on condition that it (the supplement) is of the same sign (i.e. ascending or descending) as what we are comparing. The passage 94:14-15 is completely parallel, but deals with descent. — **94:16** *occurs at*: Translate ' results from ' (cf. the next line). — **94:17-18** Translate: ' the only thing that can result is *JS*, and ḤS does not result from the difference between AḤ and *JS*', with the consequential changes in the Arabic text. This is merely a statement of the equation: (HA + HJ) — (ḤA + JS) = ḤS. — **96:6** ***He added it to its true longitude***: Cf. 79:18, with Kennedy's note. The $\frac{4}{25}$ of the equation here is ' the magnitude of the transit ' (80:8). It is this which is added to the true longitude. — **96:13-14** Translate: ' the five planets have in common what is necessary: one of the two (necessary things) is a deferent and the other an epicycle '. — **97:12 ff.** The apparent confusion of this passage can nearly all be resolved by elucidation of the meaning of the word *minṭaqa* and improvement of the translation. The *minṭaqa* (literally ' belt ', see my note on 5:16) is encountered by the planet only at 6 and 12 signs, i.e. at perigee and apogee (whether of epicycle or eccentric). So it must be used by Māšāllāh to mean ' the diameter drawn between apogee and perigee ', the same as the ' first diameter ' in 97:4. I know of no parallel to this use. Now translate as follows: 97:15-16: ' up to four signs and a half (i.e. where its epicyclic equation is a maximum) it is falling from the *minṭaqa* downward '. In 97:17 it would be best to change *mina l-minṭaqa* to *ila l-minṭaqa*, and translate: ' ascending from its descent to the *minṭaqa* ', though the text could be kept. — **98:10** *previously mentioned*: At 92:1. Cf. my note on 92:9. — **98:17** *conjunction*: Though the word occurs twice in the translation of this line, the Arabic term is in the first place *qirān* and in the second *ittiṣālan* where it is better translated ' connection ' (see my note

on 99:10-13). ' conjunction ' implies only that the longitudes are the same. The first part of the line will then mean ' if the latitudinal difference between the two bodies in conjunction is more than one minute '. — **98:18-19** For the correct translation see my note on 99:10-13. — **99:6-8** (Note on p. 181): Neugebauer's solution for the date of the horoscope is not acceptable, as it gives a longitude for Mars of 302°, differing by 42° from the figure in the text (344°). No vernal equinox between the years —600 and +1649 yields a configuration of the three planets anywhere near that of the horoscope, so we must assume either that Māšāllāh took an imaginary position or that he made an error of calculation. — **99:10-13** Translate: ' And because the conjunction (*qirān*) is in *opposition* (i.e. Jupiter and Saturn are in conjunction with each other but in opposition to the sun) and Mars is proceeding towards connection (*al-ittiṣāl*) with *Jupiter* and the sun with Saturn '. The meaning of *ittiṣāl* here becomes clear when we note from the positions given in 99:6-8 that Mars is nearly in opposition to Jupiter and Saturn to the sun (since it is the ' year-transfer ' the latter is in Aries 0°). We might conceivably suppose that *ittiṣāl* refers to the fact that the next conjunctions that were going to take place would be between the planets mentioned. But that is not true: for Mars will be in conjunction with Saturn before it is in conjunction with Jupiter. We are therefore forced to take *ittiṣāl* to mean ' exact opposition '. This is confirmed and elaborated by 98:18-19, which we may now translate: ' he means by " connection " (*al-ittiṣāl*) transit in opposition with the sectors being different '. Similarly in 99:13 ' the two in conjunction ' should be replaced by ' the two connected ones '. — **100:6** *one degree and eight minutes*: (Note on p. 183): The e_s for 171° derived from al-Khwārizmī's *zīj* is 0;59°, not 0;55°. This would give a result slightly closer to B.'s value, but agreement is still not good. In view of B'.s statement (54:12-13) that the planetary parameters in the *Shāh Zīj* and al-Khwārizmī were identical, which is corroborated by the information we can derive about the former and the printed text of the later version of the latter (compare my Table II above with the table printed on p. 178 of the commentary), we must assume that B. has in error taken the value for *169*;52° instead of 170;52° here. al-Khwārizmī's table gives 1;7° for this (B. has 1;8°). — **102:2-4** Translate: ' As for the descent of Mars, that is (found) *with respect to* its position from the apogee, not from the position in which he placed Jupiter; but *in that respect* (i.e. distance of Mars from the apogee) it is ascending, and subtraction, not addition, is required (reading *munīya* for *mn*) to get the elevation ', i.e. Māšāllāh's operation in 101:19 to get the elevation of Jupiter above Mars should have been one of subtraction, not addition. — **102:5-11** The big discrepancies which emerge between B.'s calculations in this passage and those of the commentator (p. 186), who uses al-Khwārizmī's *zīj*, are probably to be explained by the fact that B. was computing with the *adjusted* centre (as explained on p. 183 of the commentary), while Kennedy uses the unadjusted centre. Though the 3;15° of the text at 102:8 is wrong on any supposition (perhaps emend to 7;15°?) the suggested emendation of 7;30° to 9;30° at 102:9-10 is certainly wrong, for B. states at 102;12 that *all*

his results for Mars are far from Māšāllāh's, whereas 9;30° is very close indeed to Māšāllāh's 9;26° (101:15). — **102:19-103:1** The reference is to 102:2-4. — **103:1-3** These statements about the sun and Saturn are true only if both are now considered to be in the actual position of the sun, namely Aries 0°. — **103:8** *worked backward on it*: Perhaps rather: ' worked them (similar calculations to the above) out all over again for the end of the two hundred and forty years '. The two hundred and forty
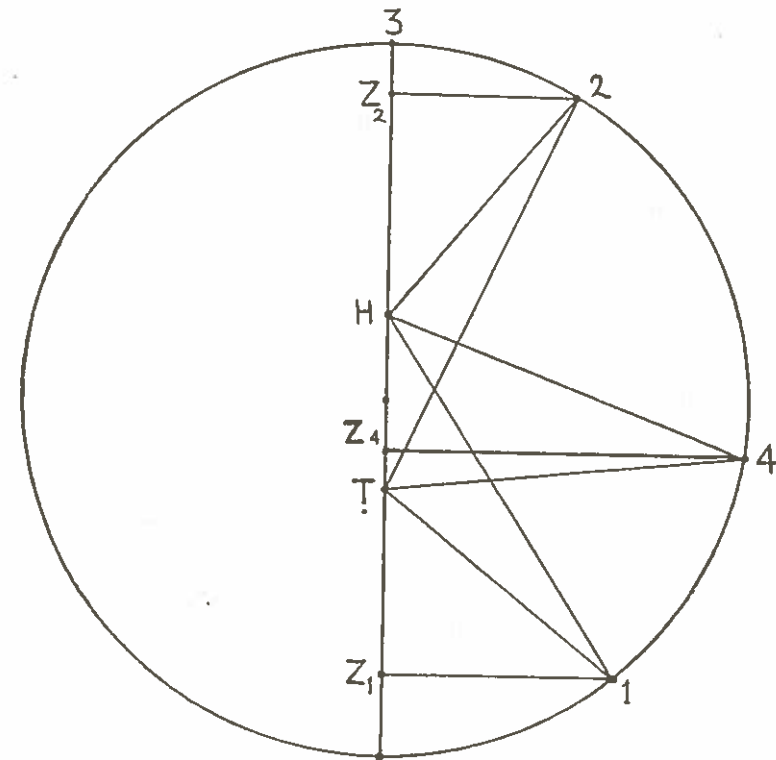
Fig. VI

years are the period of 12 mean conjunctions of Jupiter and Saturn (12×20 years) during which the phenomenon moves from one triplicity to the next (see Kennedy's note on p. 126). — **105:7-9** Four possible situations are described, which are depicted in my Fig. VI: (1) Form ZṬ + ṬH; (2) Form ZṬ — ṬH; (3) B is at apogee or perigee, so there is no question of adding or subtracting; (4) Form ṬH — ZṬ (when 'the eccentricity is partitioned '). — **106:4** The ' small circle ' is probably not one of latitude, but of declination (the two planets being supposed to have the same longitude). — **106:5-7** Translate: ' And, they being in

the condition of equality, if one of them is at the extreme of its latitude and the other increasing in latitude, then there is no doubt that the one increasing is disposed to elevation ', i.e. if the two planets have equal latitude, but planet A is at its maximum while B has not yet reached it, then B will (in the future) be elevated above A. — Kennedy's **Bibliography**: p. 190 No. 22 This is a muddle between two quite different books. It should read: Ptolemy, Claudius, The Handy Tables, ed. and transl. by Halma as: Commentaire de Théon d'Alexandrie sur les Tables Manuelles Astronomiques de Ptolemée, 3 parts, Paris, 1822, 1823, 1825.

Bibliography

Āryabhaṭīya: The Āryabhaṭīya of Āryabhaṭa, translated with notes by Walter Eugene Clark, Chicago, 1930.

Bidez-Cumont: Les Mages Hellénisés, par Joseph Bidez et Franz Cumont. 2 tomes. Paris, 1938.

al-Bīrūnī, Chronology: The Chronology of Ancient Nations of Albêrûnî, translated and edited by Dr. C. Edward Sachau, London, 1879.

— Chronology (Arabic text): Chronologie orientalischer Völker von Albêrûnî, herausgegeben von Dr. C. Eduard Sachau, Leipzig, 1878. (Reprinted Leipzig 1923 and Baghdad 1963).

— India: Alberuni's India. An English Edition by Dr. Edward C. Sachau. 2 vols. London, 1888. (Reprinted London 1910).

— Tafhīm: The Book of instruction in the elements of the Art of Astrology by al-Bīrūnī. The translation by R. Ramsay Wright. London, 1934.

Boilot: L'Oeuvre d'al-Beruni, Essai Bibliographique par D. J. Boilot, O.P. Mélanges de l'Institut Dominicain d'Études Orientales du Caire 2 (1955) pp. 161-256.

Bouché-Leclercq: L'Astrologie Grecque, par A. Bouché-Leclercq. Paris, 1899. (Reprinted Brussels 1963).

Brockelmann: Geschichte der Arabischen Litteratur von Carl Brockelmann. Zweite Auflage. 2 Bde. (1943, 1949). 3 Supplementbde. (1937, 1938, 1942). Leiden.

al-Farġānī: Muhamedis Alfragani Arabis Chronologica et Astronomica Elementa, auctore M. Iacobo Christmanno. Frankfurt, 1590.

Firmicus: Firmicus Maternus Matheseos Libri VIII edd. W. Kroll, F. Skutsch, K. Ziegler. 2 voll. Leipzig (Teubner) 1897, 1913.

Handy Tables: Commentaire de Théon d'Alexandrie sur les tables manuelles astronomiques de Ptolemée traduites ... par M. l'Abbé Halma etc. 3 parties. Paris, 1822, 1823, 1825.

Kennedy [1]: E. S. Kennedy, The Sasanian Astronomical Handbook Zij-i Shāh. Journal of the American Oriental Society vol. 78 (1958) pp. 246-262.

Kennedy [2]: E. S. Kennedy, A Survey of Islamic Astronomical Tables. Transactions of the American Philosophical Society N.S. Vol. 46 pt. 2 (1956) pp. 123-177.

Khaṇḍakhādyaka: The Khaṇḍakhādyaka, an astronomical treatise of Brahmagupta, translated into English ... by Prabodh Chandra Sengupta. Calcutta, 1934.

al-Khwārizmī: Die Astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī in der Bearbeitung des Maslama ibn Aḥmed al-Madjriṭi und der latein. Übersetzung des Athelhard von Bath ... herausgegeben von H. Suter. Kgl. Danske. Vid. Selsk. Skr. 7 R. Hist./filos. Afd. III 1. Copenhagen 1914.

Kirfel: W. Kirfel, Das Purāṇa vom Weltgebäude. Bonner Orientalistische Studien N.S. 1. Bonn, 1954.

Nazim: M. Nazim, A Unique Manuscript of Astronomy. Archaeological Survey of India, Annual Report 29 (1929-30) pp. 232-3.

Neugebauer [1]: O. Neugebauer, The transmission of Planetary Theories in Ancient and Mediaeval Astronomy. Scripta Mathematica 22 (1956) pp. 165-192.

Paulus Alexandrinus: Paulus Alexandrinus ΕΙΣΑΓΩΓΙΚΑ ed. Ae. Boer. Leipzig (Teubner) 1958.

Philoponus: Ioannes Philoponus de Aeternitate Mundi contra Proclum ed. Hugo Rabe. Leipzig (Teubner) 1899.

Pingree [1]: David Pingree, Astronomy and Astrology in India and Iran. Isis 54 (1963) pp. 229-246.

Ptolemy, Almagest: Claudii Ptolemaei Opera quae exstant omnia. Vol. I Syntaxis Mathematica ed. J. L. Heiberg. 2 partes. Leipzig (Teubner) 1898, 1903.

— Manitius: Des Claudius Ptolemäus Handbuch der Astronomie ... übersetzt ... von Karl Manitius. 2 Bde. Leipzig (Teubner) 1912-13. (Reprinted Leipzig 1963).

— Opera Minora: Claudii Ptolemaei Opera quae exstant omnia. Vol. II Opera Astronomica Minora ed. J. L. Heiberg. Leipzig (Teubner) 1907.

— Tetrabiblos: Claudii Ptolemaei Opera quae exstant omnia. Vol. III i ΑΠΟΤΕΛΕΣΜΑΤΙΚΑ edd. F. Boll. et Ae. Boer. Leipzig (Teubner) 1957.

Rizvi: S. S. H. Rizvi, A Unique and Unknown Book of al-Beruni, Ghurrat-az-Zijat or Karana Tilaka. Islamic Culture 37 (1963) pp. 112-130, and following numbers of the same journal.

Rosen: The Algebra of Mohammed ben Musa, ed. and tr. by Frederic Rosen. London, 1831.

Steinschneider: Moritz Steinschneider, Die Arabischen Übersetzungen aus dem Griechischen. Graz, 1960.

Sūryasiddhānta Burgess: Translation of the Sûrya-Siddhânta by Rev. Ebenezer Burgess, reprinted from the edition of 1860. Calcutta, 1935.

(The original edition is in the Journal of the American Oriental Society 6 (1860) pp. 141-498).

Sūryasiddhānta Shukla: The Sūrya-Siddhānta with the commentary of Paramesvara, edited by Kripa Shankar Shukla. Hindu Astronomical and Mathematical Texts Series No. 1. Lucknow, 1947.

Toomer [1]: G. J. Toomer, A Note on Tamil Astronomical Tables. Centaurus 9 (1963) pp. 11-15.

Vettius Valens: Vettii Valentis Anthologiarum libri ed. G. Kroll. Berlin, 1908.

Zaehner Dawn and Twilight: R. C. Zaehner, The Dawn and Twilight of Zoroastrianism. London, 1961.

— Zurvan: R. C. Zaehner, Zurvan, A Zoroastrian Dilemma. Oxford, 1955.

بإذن من جامعة بيروت الأمريكية

إعادة طبعة بيروت ١٩٥٩م

طبع في ١٠٠ نسخة

نشر بمعهد تاريخ العلوم العربية والإسلامية
بفرانكفورت - جمهورية ألمانيا الاتحادية
طبع في مطبعة شتراوس، مورلنباخ، ألمانيا الاتحادية